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Constrained Quadratic Design of FIR Data-Communication Filters with Linear Matrix Inequalities

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Abstract— **An example design of the nonlinear-phase FIR portion of a hybrid analog/digital data-communication receive filter is presented in which mean-square adjacent-channel interference (ACI) is minimized subject to constraints on Gaussian noise power and mean-square intersymbol interference (ISI) using linear matrix inequalities (LMIs).**

I. Introduction

Practical filter design is most powerful when it is done in-context, optimizing the performance of part or all of a system directly (through the filter) rather than optimizing the filter separately based on specifications such as passband/stopband ripple. Thus an FIR filter in a communication system might be designed to minimize common sources of system error, such as intersymbol interference (ISI), adjacent-channel interference (ACI), and Gaussian noise, instead of approximating (in an equiripple or other sense) some ideal function. The key to formulating such problems is to develop a framework for mapping general system constraints to numerical constraints, such as linear matrix inequalities (LMIs), that are usable by an optimization engine.

Such a framework is provided by the companion paper [1], which shows how quadratic constraints on FIR filters embedded in a multirate DSP system can be mapped to LMIs, thus allowing the design problem to be solved with powerful, global optimization engines such as *LMITOOL* [2], which is included in Scilab and available for Matlab. This paper illustrates the power of such techniques by designing the FIR portion of an example hybrid continuous-/discrete-time receive filter to minimize mean-square ACI while constraining mean-square ISI and Gaussian noise power.

For an overview of LMI methods for general-purpose FIR filter design, see Wu *et al.* [3] and Balakrishnan *et al.* [4]. Settle *et al.* [5] use LMIs to address communication-filter optimization with H_2 and H_∞ methods.

II. Problem Formulation

The dashed-box baseband-equivalent communication system in the hypothetical test system of Fig. 1 includes a transmitter, ACI/Gaussian-noise channel, and pre-decision receive filter. The lower path provides uncorrupted reference symbols for comparison. System parameters and filters except FIR filter h_k are assumed to have been chosen earlier on suitable general principles. Shaded blocks represent responses determined by the optimization. Overall error power P_{err} is here viewed as the sum of three independent error powers P_{ACI}, P_{ISI}, and P_n. FIR filter response h_k will be optimized by minimizing P_{ACI} while constraining the other two with LMIs.

The analog components of Fig. 1 can be accommodated by incorporating them into an equivalent, purely discrete-time system as shown in Figure 2. Notationally, function $h * g$ is the convolution of functions h and g, and $(h * g)(t)$ is the value of that function at time t (similarly in discrete time). Processes α_k and η_k (stationarity makes τ unnecessary) represent ACI and noise after prefiltering and sampling. This discrete-time system can then be decomposed to allow the independent measurement of the three errors of interest, as shown in Fig. 3. The ISI level is determined by a single-rate equivalent response. The invariance of ACI and noise power to decimation allows them to be measured before decimation.

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III. A Numerical Example

In this example a root raised-cosine transmit pulse shape is assumed, with 35% excess bandwidth and a signaling interval of T. Adjacent channels are centered at $\pm 1.5/T$. The analog portion of the receive filtering, with frequency response $G(f)$, is a third-order Butterworth lowpass filter with a cutoff frequency of $1/(2T)$. Magnitude $|G(f)|$ roughly approximates the desired root raised-cosine shape, which lessens the requirements on the FIR filter. The analog-filtered signal is sampled at $M = 4$ times signaling rate $1/T$ prior to FIR filtering and then decimated by M. The nonlinear-phase FIR filter has 13 real coefficients h_k, which will be optimized. Sampling epoch τ is chosen here as the sum of the DC group-delay of the analog filter and half the FIR filter length, a choice that could likely be improved upon.

The optimization of the FIR filter in this example design is procedurally specified in Table 1 using a formal system of operators [1]. The specification includes four constraints, which are mapped to LMIs according to the formalism. System gain $(u * h)(0)$, which scales the data symbols, is first constrained to unity. Normalized ISI power P_{ACI}/P_ρ is then limited to 1% rms. Actual (mismatched) output noise power P_n is normalized by ideal (matched) output noise power $P_{n\text{opt}}$ and limited to a mismatch penalty of 0.05 dB. (If $n(t)$ is white with spectral density N_0, then $P_{n\text{opt}} = N_0/\|Tp\|^2$.) Finally, minimizing an auxiliary variable while using it to limit normalized ACI power effectively minimizes the latter while keeping the objective linear, as required by our semidefinite-programming engine. The resulting optimized value of P_{ACI}/P_ρ is -47.0 dB.

The equivalent system pulse $\sum_k h_k(Tp*g)(t+\tau-kT/M)$ is shown in Fig. 4. Figure 5 shows the key frequency responses.

References

[1] Jeffrey O. Coleman, "Systematic mapping of quadratic constraints on embedded FIR filters to linear matrix inequalities," in *Proc. Conference on Information Sciences and Systems*, Princeton, Mar. 1998.

[2] L. El Ghaoui, F. Delebecque, and R. Nikoukhah, *LMI-TOOL: A User-friendly Interface for LMI Optimization*, ENSTA/INRIA, 1995, Software available via anon. FTP from `ftp.inria.fr:pub/elghaoui/lmitool`.

[3] Shao-Po Wu, Stephen Boyd, and Lieven Vandenberghe, "FIR filter design via semidefinite programming and spectral factorization," in *Proc. IEEE Conf. on Decision and Control*, 1996, pp. 271–276.

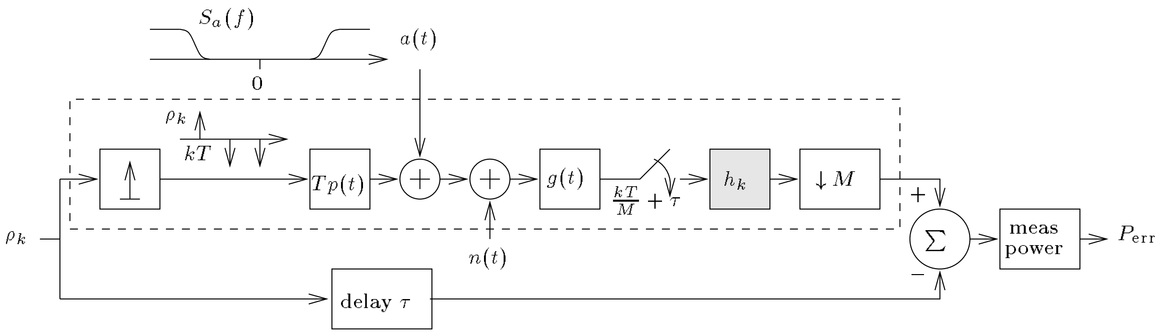

Fig. 1. This communication test system allows the measurement of ISI, ACI and Gaussian noise powers. The factor of T on the transmit pulse leads to a convenient normalization.

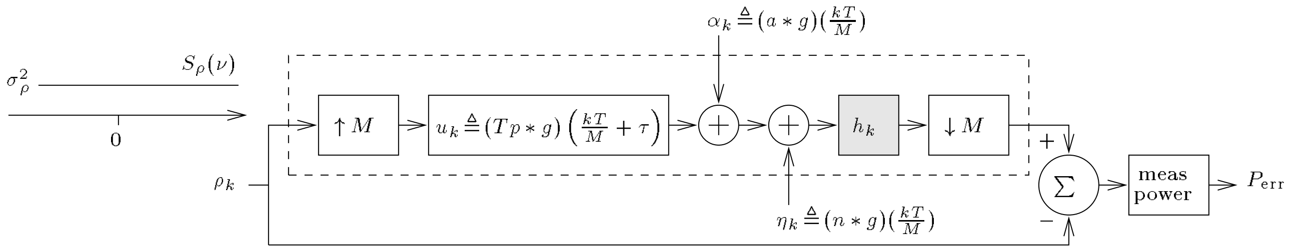

Fig. 2. A multirate, all discrete-time system which is equivalent to the hybrid system of Fig. 1. Frequency variable ν is normalized to the sampling rate.

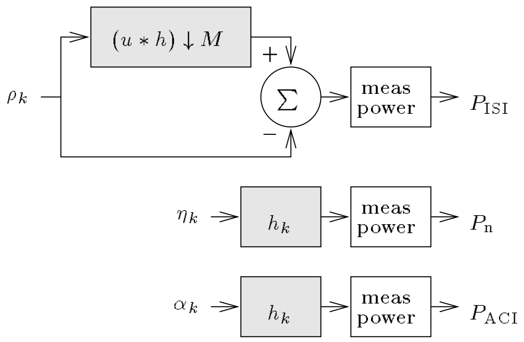

Fig. 3. This system measures ISI, ACI, and noise power independently, using three single-rate equivalent systems.

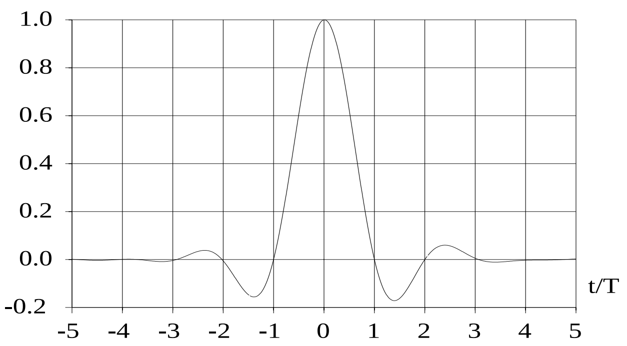

Fig. 4. Optimized equivalent system pulse.

ρ has spectrum $S_\rho(\nu)$
η has spectrum $S_\eta(\nu)$
α has spectrum $S_\alpha(\nu)$

$\mathbf{h} = \mathbf{newSequence}(13)$
$w = \mathbf{convSequence}(\delta)$
$P_\rho = \mathbf{power}(\rho)$
$P_{\mathrm{ISI}} = \mathbf{power}(\rho * ((u*\mathbf{h})\downarrow M - w))$
$P_n = \mathbf{power}(\eta * \mathbf{h})$
$P_{nopt} = N_0/\|Tp\|^2$
$P_{\mathrm{ACI}} = \mathbf{power}(\alpha * \mathbf{h})$

$\gamma = \mathbf{newVariable}(\,)$
minimize γ
subject to

$$\begin{aligned} (u*\mathbf{h})(0) &= 1 \\ P_{\mathrm{ISI}}/P_\rho &< 10^{-4} \\ P_n/P_{nopt} &< 10^{0.05/10} \\ P_{\mathrm{ACI}}/P_\rho &< \gamma^2 \end{aligned}$$

Table 1. Procedure to optimize FIR filter h_k of Fig. 3 to meet system specifications.

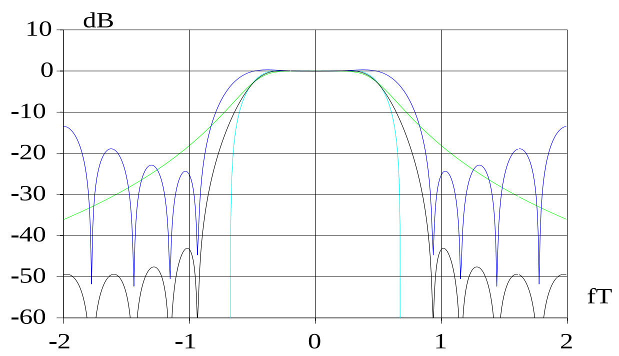

Fig. 5. The key frequency responses of the numerical example, on the left edge moving down: FIR response $H(fT/M)$, analog response $G(f)$, and optimized cascade response $H(fT/M)G(f)$, with ideal response $P(f)$ in the center.

[4] V. Balakrishnan and L. Vandenberghe, "Linear matrix inequalities for signal processing: An overview," in *Proc. Conference on Information Sciences and Systems*, Princeton, Mar. 1998.

[5] T. Settle, M. Zoltowski, and V. Balakrishnan, "Optimal multi-channel equalizers based on linear matrix inequalities," in *Proc. First Annual UCSD Conference on Wireless Communications*, UC San Diego, Mar. 1998.