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Signals and Systems II

Part VI: Vestigial-sideband modulation and nonintegral rate changes

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This six-part series is a mini-course, focused on system concepts, that is aimed at the gap between Signals and Systems and the usual first DSP course.

Part V discussed frequency conversion and its use in larger systems. This sixth and last article in the series is about two topics: first, vestigial-sideband modulation as a example system of significant complexity and, second, changing a sample rate by a rational but nonintegral ratio. Figures are numbered in one sequence across the entire series, and gaps appear in their numbering in some individual articles. Figures are posted for instructional use on the author's website: *http:// alum.mit.edu/ www/ jeffc*

PART VI

Vestigial-Sideband Modulation

We have seen how the redundancy of the real and imaginary parts of one particular type of complex signal, an analytic signal, can be exploited. Over and over in systems we have added the conjugate to remove the redundant imaginary part, which we then later restored. With the redundancy removed the signal was purely real and therefore more easily realized in an analog setting. It is interesting, however, that while removing one form of redundancy, the addition of the conjugate always introduced another, a spectral redundancy in which the positive- and negative-frequency halves of the spectrum were each other's conjugate mirrorings.

Consider the system of Figure 23 from Part III, reproduced here. The channel signal shown displays conjugate symmetry, a form of spectral redundancy with the useful property of making the channel signal real. But what if the baseband signal were real also? The associated conjugate symmetry of the baseband spectrum would then result in yet another sort of spectral redundancy in the channel signal, one that would usually be without purpose other than to give the system a new and quite standard name: *double-sideband (DSB) modulator/demodulator*, with "double" referring to the baseband spectral redundancy.

The bandwidth occupied by this overredundant channel signal could be halved if somehow we could remove the spectral redundancy at the modulator's baseband input and then restore it at the demodulator's baseband output. We cannot go quite this far, but it turns out that we can reversably remove all but a *vestige*, a remnant or remainder, of this spectral redundancy. We will approach this task by reversing our usual sequence to first filter and then add the conjugate.

We begin with a trivial filter. In the left column of Fig. 34 a real input signal is filtered with a frequency response that is nothing more than a uniform gain of $1/2$ at all frequencies. It halves the signal. The real filter output equals its conjugate, which when added then simply undoes the earlier halving. Of this "filter" we can say only that the system output signal is the same as the system input.

In the middle column in Fig. 34, the filter frequency response is *conjugate asymmetric:* it and its conjugate mirroring are negatives of each other. The spectrum of the filter's output signal has the same property (why?), and so adding the conjugate results in complete signal cancellation; the output is zero.

In the rightmost column, the filter frequency response is the sum of the frequency responses from the first two columns, and the filter output is likewise the sum of the filter outputs from the first two columns; it equals the original input signal. And here we see the key idea: begin with a frequency response that is constant and add a conjugate-asymmetric frequency response chosen to cancel the constant over some portion of the signal bandwidth.

The resulting filter is an ideal *vestigial-sideband (VSB) filter;* it removes part of the signal spectrum in such a way that addition of the conjugate will restore it. The idea is generalized in Fig. 17 to the nonideal case in which there is a net effect, presumably desired, on the real input signal. This net effect is that of an equivalent filter with a frequency response obtained by beginning with that of the actual filter and adding its conjugate.

It is tempting to attempt to fit this scheme for redundancy elimination and restoration into the Fig. 30 state diagram, starting with a real signal at the bottom of the diagram. But as drawn the diagram would be overrestrictive in this application, because it requires the complex signal produced through filtering to be analytic. VSB filters generally produce signals that are somehow not quite analytic—signal and conjugate do overlap spectrally in some small and controlled way—so we must again augment the diagram. The new state diagram, in Fig. 33, puts the new states on the bottom and gives the obvious name to the new modulation scheme.

In Fig. 34 a discrete-time (d.t.) input signal was shown, and indeed

VSB modulation is often used for data transmission by coding data into the real baseband input samples. Nothing prevents VSB use in analog systems, however, and VSB-modulated video signals have indeed been in homes for half a century in analog television signals.

Another special case of historical interest results from modifying the leftmost column of Fig. 17 by removing the red ticks to make signal and filter analog and then using an input spectrum that is essentially zero in the filter's transition band around zero frequency. This makes the filter output analytic, and the adjective VSB is then replaced with *single sideband (SSB)*. Classically SSB was used for speech signals. because speech can be filtered to meet these input requirements and remain quite intelligible.

Communication-systems textbooks traditionally discuss SSB modulation, even though applications are very rare today. The interested reader can work out the details independently by beginning with Fig. 17 as modified in the previous paragraph, splitting the filter into two filters with identical passbands and stopbands—making them square roots is tempting but unnecessary—and then inserting an inphase and quadrature (IQ) modulator, a channel, and an IQ demodulator between the two filters.

As constructed this would be a *phasing-method* SSB system, and these were occasionally built in the mid-20th century. The *filter method* of SSB generation was far more common however: the two baseband filters are eliminated in favor of passband equivalents immediately adjacent to the channel, and this leaves the baseband signal purely real so that the IQ modulator and demodulator can be replaced with the simpler DSB modulator and demodulator.

It is for data transmission that VSB modulation is being designed into systems today, however. An interesting and common special case is considered next.

VSB for Data: The Offset-QPSK Case

In Fig. 35 we follow the evolution of a system idea through five stages, the first three of which appear stacked in the leftmost column. At the upper left, we begin with basic Nyquist signaling as per Fig. 14. A real d.t. input is Nyquist filtered to create an analog signal and then sampled to restore it to its original form. Moving down, the second system shows how that Nyquist filter can be obtained from a VSB filter by adding the conjugate. The bottom sketch is a baseband VSB signaling system of the Fig. 17 sort.

The frequency response of this particular VSB filter was given conjugate symmetry about its midpoint. This feature is not required for VSB but enables the strategy used in the next version of this particular system, shown in the middle column.

To derive the latter, immediately follow the VSB filter at the lower left with a pair of frequency shifts, first to the left and then to the right, each by a quarter of the signaling rate. Then move the filter through the first shift, shifting its response to the left appropriately, to place it between the two frequency shifts. This centers the filter response about the origin, so the conjugate symmetry about its midpoint now gives it a real impulse response, an implementation convenience.

Now split the filter into the two root-VSB filters in the pink-shaded area in the middle column. The center yellow section can now be inserted between the two filters to modulate the signal to passband and demodulate it back, using the second root-VSB filter to eliminate the conjugate component, much as was done in the IQ modulator and demodulator on the left in Fig. 23 from Part III, reproduced here.

Steps are rearranged and grouped on the right into a pair of frequency shifts surrounding an IQ modulator and an IQ demodulator, each with root-VSB spectral shaping and each subject on its own to any of the design alternatives we have discussed.

This modulator/demodulator pair is, remarkably, just as well suited to Nyquist signaling, but for d.t. signals at half the input sampling rate shown. The VSB filter meets the half-rate Nyquist criterion because of the extra symmetry condition imposed on the VSB filter above. In the Nyquist-signaling application, the data samples could be complex instead of real as in this VSB system, so halving the sampling rate would cause no loss of data-carrying capacity. Either the complex-input Nyquist system or the present real-input VSB system might be preferable in a given context, but only for subtle reasons beyond the scope of this discussion.

It is the frequency shifts at input and output that make this system unique. The complex exponentials by which they multiply their inputs are at one quarter of the sampling rate and so just multiply the samples by j^n and j^{-n}, effectively multiplying even-indexed samples by ± 1 and odd-indexed samples by $\pm j$. The input to the first frequency shift is real, so its output comprises interlaced streams of real and imaginary samples, each at half the original rate. We can therefore visualize this first frequency shift as a commutator that steers data alternately into real and imaginary "input ports" of the IQ modulator. The signs of alternate samples of each are reversed as well.

The output frequency shift and addition of the conjugate combine to effect alternate-sample sign inversion and decommutation. To see this, recall that adding the conjugate amounts to taking twice the real part. But this follows the frequency shift's multiplication of even-indexed samples by ± 1 and odd-indexed samples by $\pm j$. The effect is that adding the conjugate steers to the output the real part of even-indexed frequency-shift inputs and the imaginary part of odd-indexed frequency-shift inputs. Alternate-sample sign inversions are implied as well and undo those imposed by the input frequency shift. Of course if the samples simply encode arbitrary data, there is certainly no harm in omitting the sign inversions from both input and output, and this is quite commonly done.

If the real data are binary, this scheme is called *offset QPSK, OQPSK*, or occasionally *staggered QPSK*. Traditionally, "offset" and "staggered" refer to the time offset of half a signaling interval between the real and imaginary baseband data samples. It would be just as logical, however, to take "offset" to refer to the frequency offset that relates the system's filtering to VSB filtering.

A Faux-Sampling Operation

Here we revisit an earlier subject, that of factoring a sampling operation into high-speed sampling and decimation. We used this technique, followed by application of the most noble identity, in the design of IQ demodulators for a carrierless-QAM modem in Fig. 22 of Part III and then for unspecified wideband and narrowband applications in Part IV's Figs. 26 and 27 respectively. Though it was not emphasized at the time, the design process in each case was simplified by a convenient ratio of input-band center frequency to desired output sampling rate. Here, we pose the problem again but with a particularly inconvenient ratio and explore one solution to the newly created dilemma.

The input signal in the top half of Fig. 36, presumably the output of an IF filter in some receiver of which this system is a part, is a real bandpass signal with 2 MHz of information-carrying bandwidth on the positive frequency axis centered at 15 MHz. It is embedded in the center of an 18 MHz band of unwanted noise. To obtain a complex baseband signal sampled at a modest rate while excluding as much noise as possible, an analytic bandpass filter with a narrow skirt width of 1 MHz and followed by sampling at 3 MHz is shown. From our previous experience we know this filter should be digital with a frequency-response period that is a multiple of the eventual output sample rate. We expect this frequency-response period to eventually become the rate of the input sampler, just as in the Part III and IV applications cited above.

In this case, however, the frequency-response period is not easily chosen. Only a 20 MHz period or those periods 40 MHz or over will permit passing the desired signal while excluding all other signal components from the passbands. It should be no surprise that the same numbers turn up if we ask at what rates we might sample this input without causing *aliasing*, causing undesired spectral components to overlap the desired one in the sampling process. Here only a sampling rate of 20 MHz or a rate of 40 MHz or more avoids aliasing. That these numbers are the same is a consequence of the most noble identity.

Now suppose—this is the twist—that A/D converters otherwise suitable for this particular application are not available that can operate at speeds 40 MHz or higher. Converters are available for 20 MHz, but 20 MHz is not a multiple of the 3 MHz output sample rate. Our entire scheme fails.

We extricate ourselves from this dilemma in the lower part of Fig. 36 by adding a first processing step that looks exactly like decimation, except that it is not applied to a d.t. input. We are applying this *faux-sampling operation* (a made-up name) to an analog signal, just as we would do if it were actual sampling. Reordering and recombination of operations later will render the apparent impracticality of this operation unimportant. The 60 MHz period of the filter frequency response is the least common multiple of the 20 MHz faux-sampling rate and the 3 MHz output sampling rate. The 60 MHz to 20 MHz ratio here is reflected in the three spectral impulses used. The lower system in Fig. 36 is then just the upper system with faux sampling added and with the frequency response made periodic.

The two systems are mathematically equivalent for inputs with the spectral structure shown, even though they are not equivalent in general. That useful compromise in generality is what this faux-sampling operation is about.

The system is transformed into a practical one in stages in Fig. 37. Omitting the input noise in the upper system for clarity, the filter is split into three stages with different response periods, and the output sampling is split into 60 MHz sampling and decimation by 20. An application of the most noble identity, not shown, has moved the 60 MHz to before the filtering.

The lower system shows the point of the faux-sampling trick: it combines the adjacent faux sampling and high-speed sampling steps to form a sampling operation at the practical rate of 20 MHz. The decimation has also been factored, using the ratios of the frequency-response periods as a guide, and the most noble identity has been used to rearrange the filtering and decimation steps.

Such a scheme can be implemented very efficiently, as implementation of the first filter can incorporate the interpolation that precedes it, and the implementation of each of the three filters can incorporate a following decimation step.

Conclusion

It should be more than clear by now, at the conclusion of this series of articles, that substantial system-configuration capability can be built on little background and using very little explicit mathematics. This author would like to see all BSEE programs include signal processing at this level. Students specializing in signal processing could then continue with the study of DSP in a more traditional framework. In addition, computer engineers comfortable with the concepts of this series would have a tremendous head start in subsequently mastering the computational side of DSP, an area likely to account for an ever-increasing proportion of their employment in coming years.

Acknowledgements

Several eager classes full of students in a senior-level Michigan Tech core course in communications catalyzed the refinement of much of this material. This series is dedicated to one of those students who was overwhelmed to tears in the final exam but pushed through to success in both the course and an engineering career.

Read more about it

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P. P. Vaidyanathan, *Multirate Systems And Filter Banks*. Prentice Hall: *http://www.phptr.com/*, 1993.

D. P. Scholnik, “Quadrature demodulation and modulation using bandpass sampling and reconstruction,” Master’s thesis, Michigan Technological University, Houghton, MI, USA, Aug. 1997.

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D. P. Scholnik and J. O. Coleman, “Simple, exact models of sample-interleaving demodulators/modulators for quadrature bandpass sampling/reconstruction,” in *Proc. 1997 Conf. on Information Sciences and Systems (CISS ’97)*, Baltimore, MD, Mar. 1997.

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J. Crols and M. S. J. Steyaert, “Low-IF topologies for high-performance analog front ends of fully integrated receivers,” *IEEE Trans. Circuits and Systems II*, vol. 45, no. 3, Mar. 1998.

About the author

Jeffrey O. Coleman (S’75–M’79–SM’99) joined the Radar Division of the Naval Research Laboratory (NRL) in Washington DC in 1978 then left it in 1985 for graduate studies, for a stint with The Boeing Company, and for a faculty position at Michigan Technological University from which he returned to NRL in 1997. His 1975/1979/1991 SBEE/MSEE/PhD degrees are from the Massachusetts Institute of Technology, Johns Hopkins University, and the University of Washington respectively, and his research is on theory and design methods in DSP. More: http://alum.mit.edu/www/jeffc

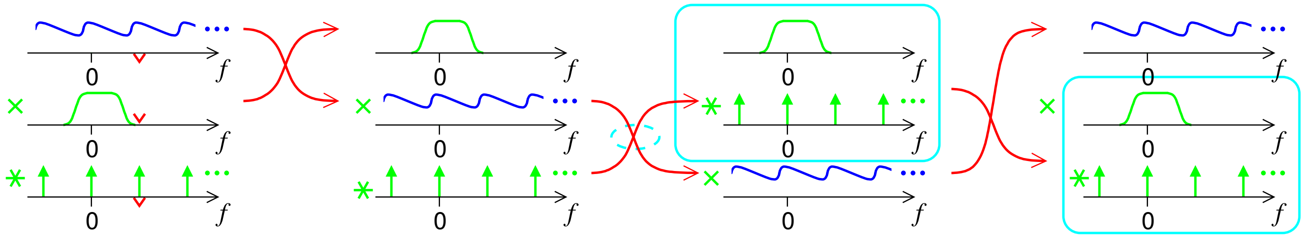

Fig. 14 (from Part II) The *most noble identity* (center) and product reordering (other red arrows) permit the top-to-bottom order of operations to be altered according to the *parenthetical groupings* and lead, on the right, to the Nyquist criterion.

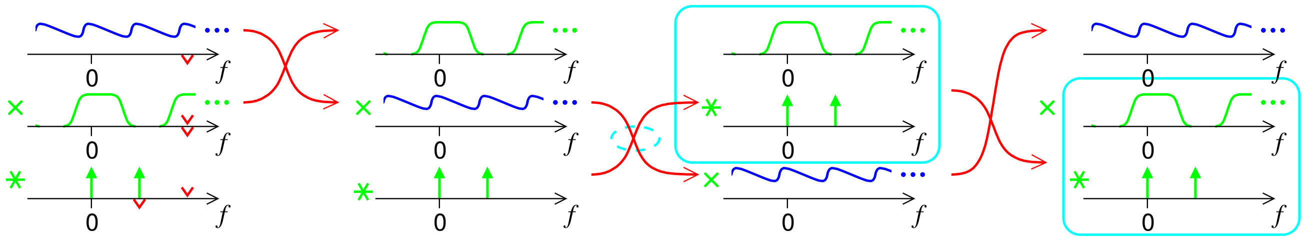

Fig. 15 (from Part IV) Only one even-numbered sample of a halfband filter's impulse response is nonzero, the sample at the origin. This derivation parallels that of Fig. 14 and in fact Nth-band and Nyquist filters are closely related.

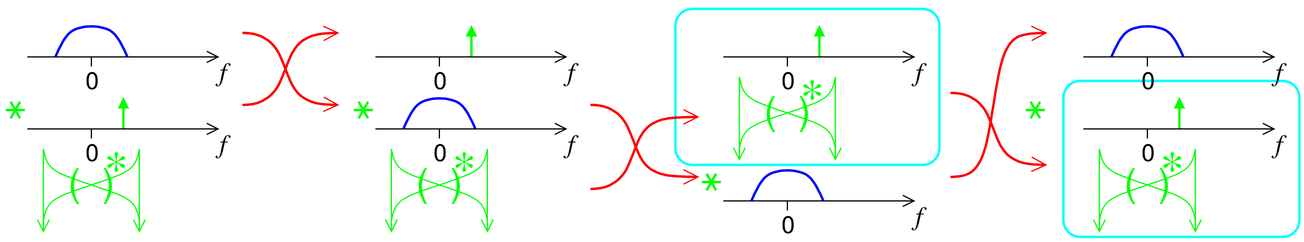

Fig. 16 (from Part V) Frequency shifting and then adding the conjugate to a real signal is equivalent to (and realized as) cosine modulation.

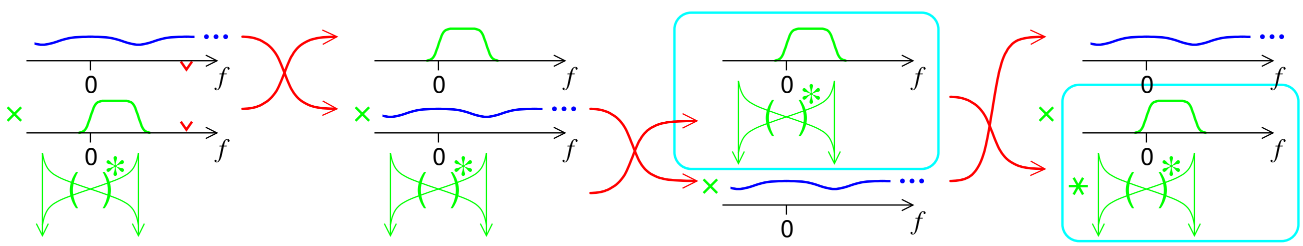

Fig. 17 Complex filtering of a real input in a VSB modulator and adding the conjugate in the demodulator yield a net real filter.

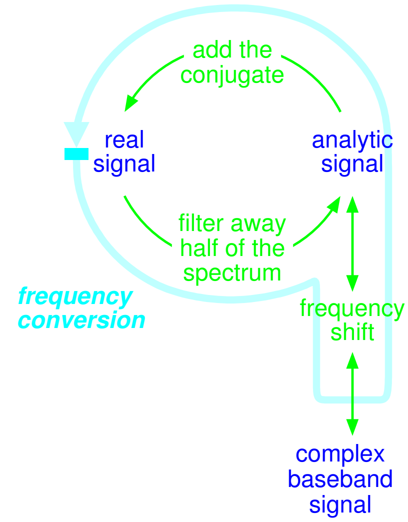

Fig. 30 (from Part V) All frequency conversions conceptually require filtering, shifting, and adding the conjugate, but the order may be varied from that shown.

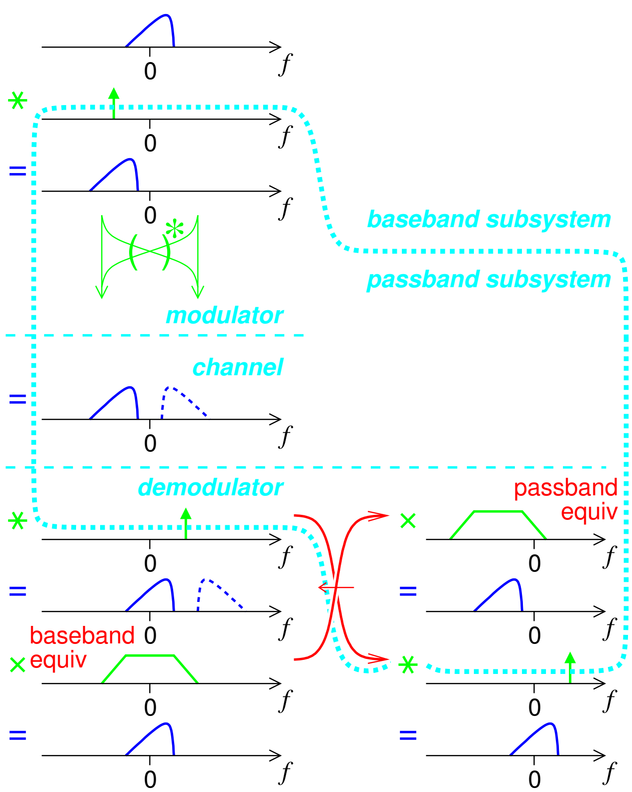

Fig. 23 (from Part III) A classic linear or IQ modulator and the usual shift-then-filter IQ demodulator on the left and an alternate, filter-then-shift IQ demodulator on the right.

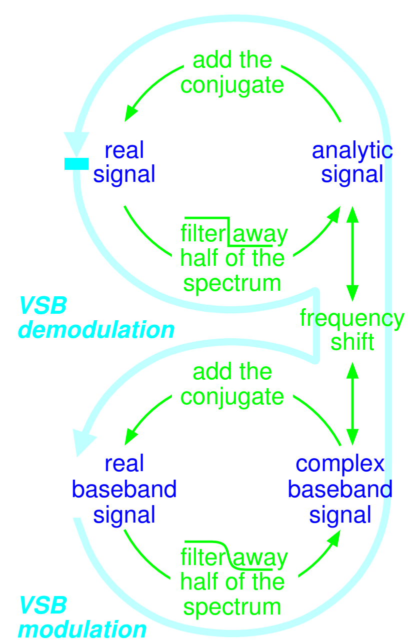

Fig. 33 In the upper and lower halves of this VSB state diagram “filter away half” is respectively in frequency only and in a combination of frequency and amplitude.

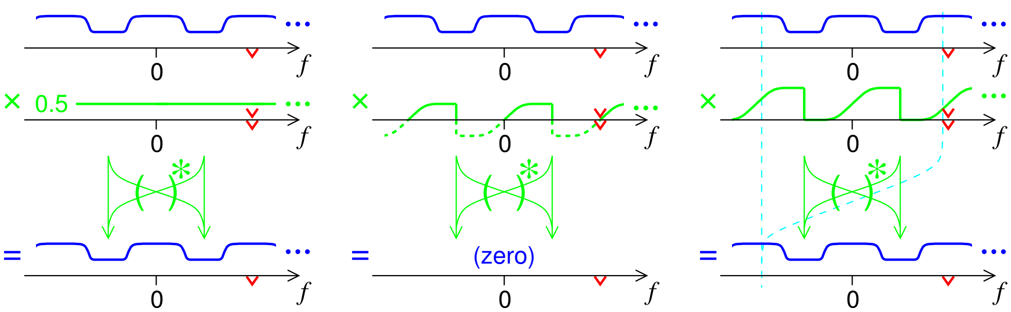

Fig. 34 Filtering a real signal and adding the conjugate using three filters: constant gain (left), conjugate-asymmetric frequency response (center), and their sum (right). The latter, a VSB filter, is used for bandwidth reduction.

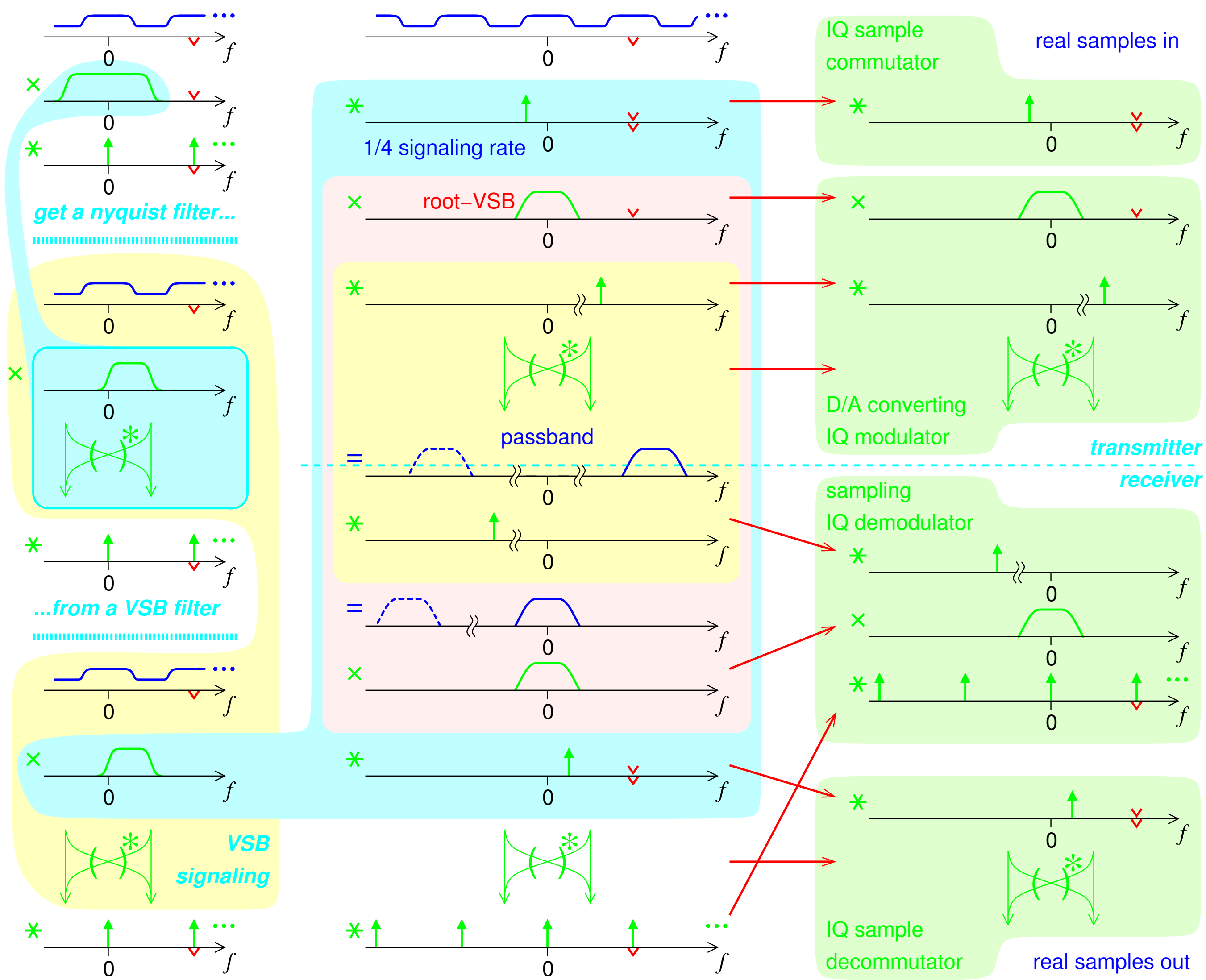

Fig. 35 On the left, splitting a Nyquist-signaling filter (top) into a VSB filter and its conjugate (center) leads to vestigial-sideband (VSB) signaling of real data (bottom). In the center, the VSB filter is obtained from a lowpass filter (blue shading) using frequency shifts of one-fourth the signaling rate. The lowpass filter is then split into root-VSB filters (pink shading) with IQ modulation and demodulation inserted inbetween (yellow shading). Steps are rearranged and grouped (green shading) on the right into an IQ modulator and demodulator with root-VSB shaping (the inside groups) and an IQ commutator and decommutator (the outside groups) that feed/fetch the real data in/out. If the real data are binary, this special case of VSB becomes offset QPSK.

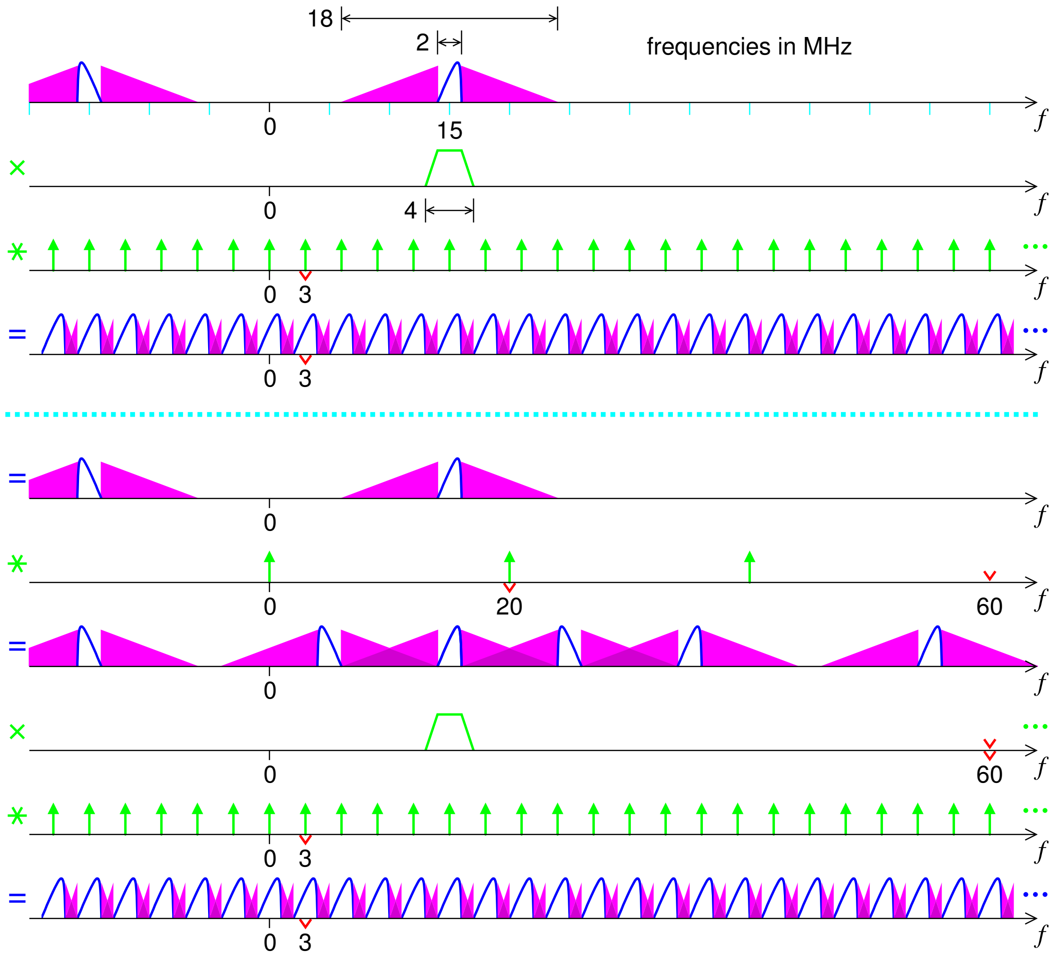

Fig. 36 The upper system shows the desired system functionality with tick marks every 5 MHz on the first axis. In the lower system a faux-sampling operation, resembling decimation but with an analog input, is inserted without harm and the filter frequency response is made periodic. (See next figure for realization.)

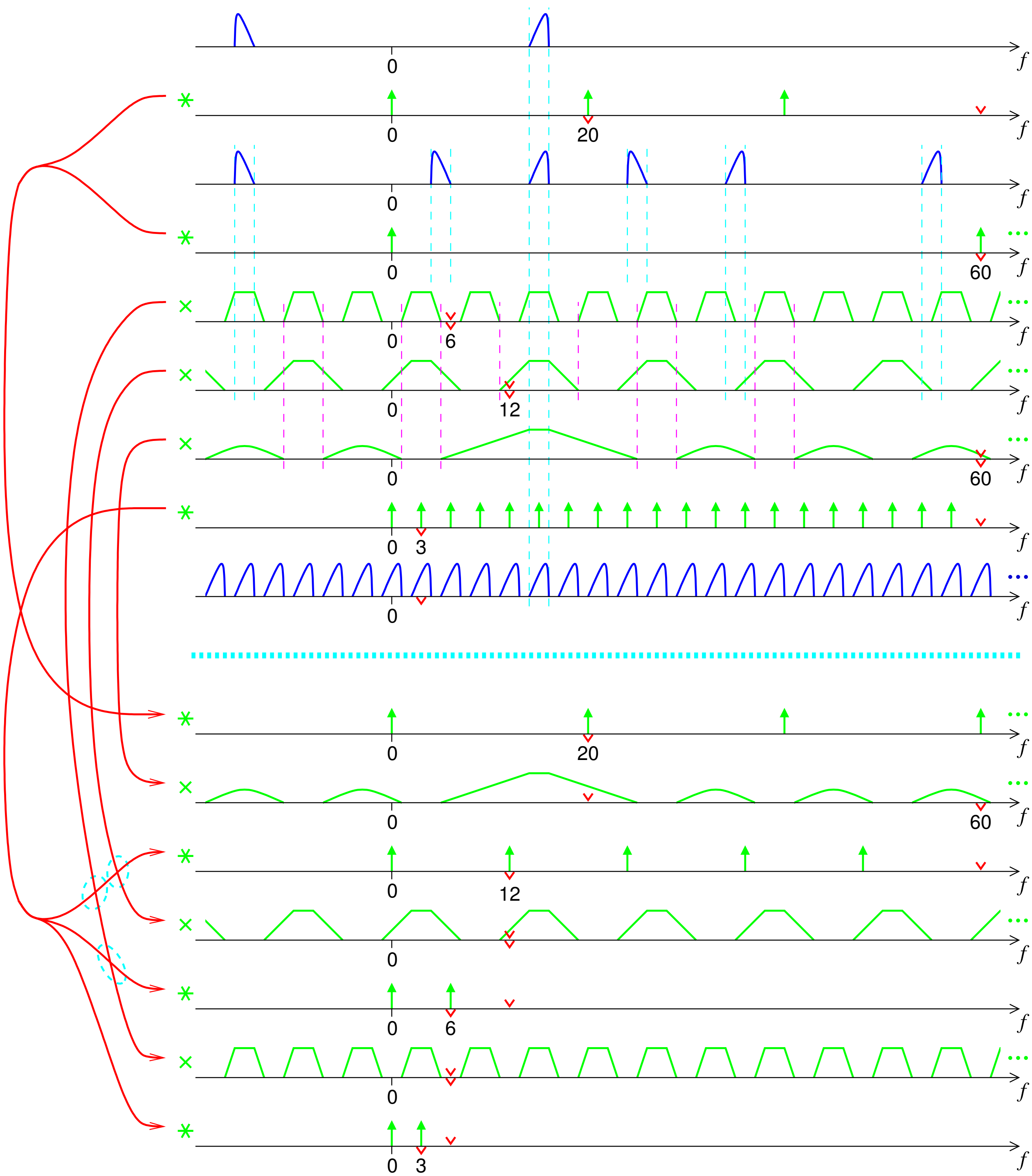

Fig. 37 To realize the system in the bottom half of the last figure, the filter is first (upper system here) split into three stages at different rates. Then (lower system here) the decimation is factored, filtering and decimation steps are reordered, and the faux sampling is combined with the high-speed sampling to produce a sampling operation at a lower, practical rate.