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Integer-Coefficient FIR Filter Sharpening for Equiripple Stopbands and Maximally Flat Passbands

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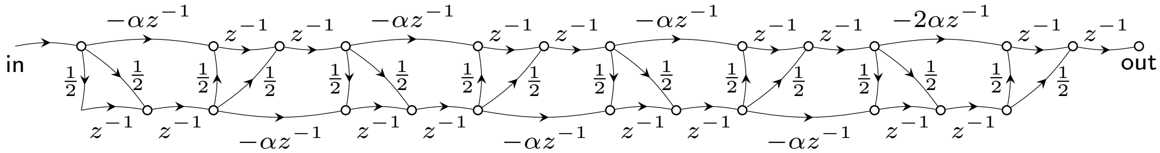
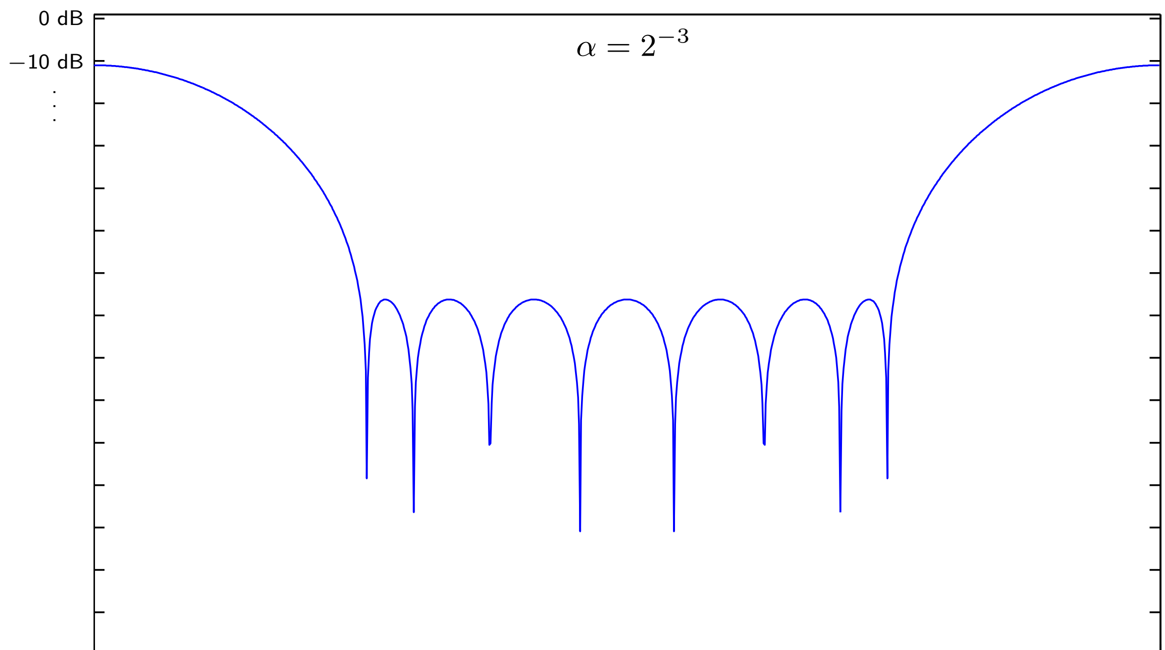
Why sharpen?

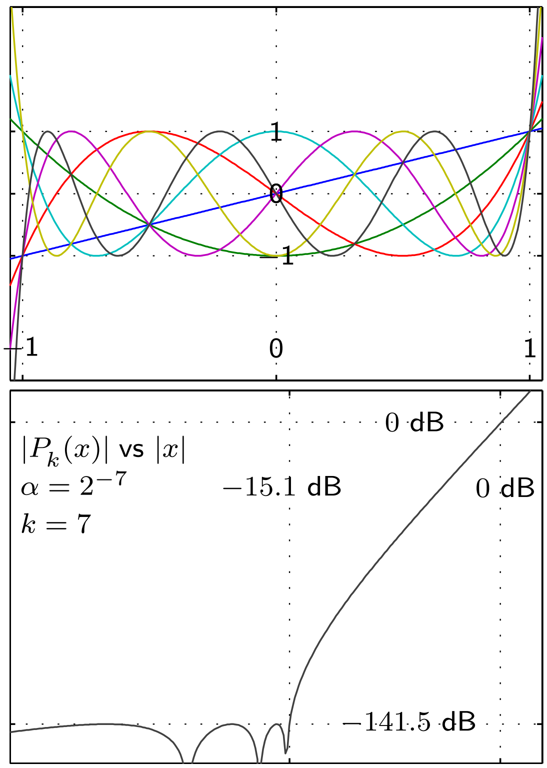
- Sharpening is an old approach that improves performance of a filter by using multiple copies it. Cascading is a special case.
- But why? Isn't an optimal FIR filter of length 100 always better than 10 copies of a length-10 filter?
- Yes. . . as long as real coefficients are used. But coefficient quantization changes everything, as sharpening can radically reduce sensitivity. (It's not clear that this was appreciated by the original developers of filter sharpening.)
- Stopbands $\gg$100 dB down are quite practical with trial-and-error designs.

How simple can an equiripple filter be?





Scale the Chebyshev polynomials of the first kind



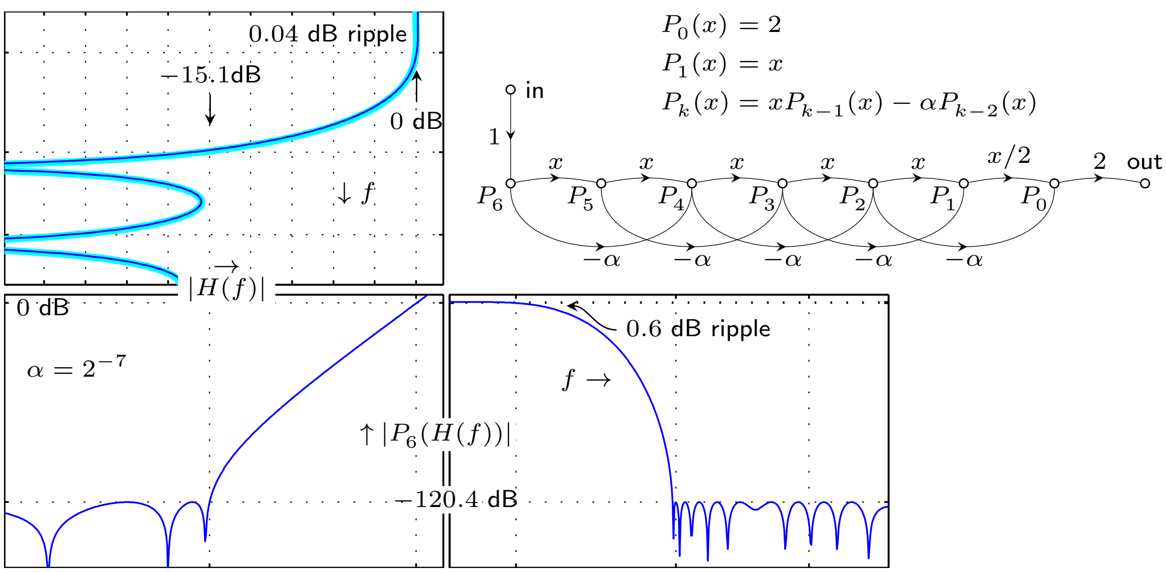
$$T_0(x) = 1$$
$$T_1(x) = x$$
$$T_k(x) = 2xT_{k-1}(x) - T_{k-2}(x)$$

$$P_k(x) \triangleq 2\alpha^{k/2} T_k(\alpha^{-1/2}x/2)$$

$$P_0(x) = 2$$
$$P_1(x) = x$$
$$P_k(x) = xP_{k-1}(x) - \alpha P_{k-2}(x)$$

Use the scaled polynomial as a transformation

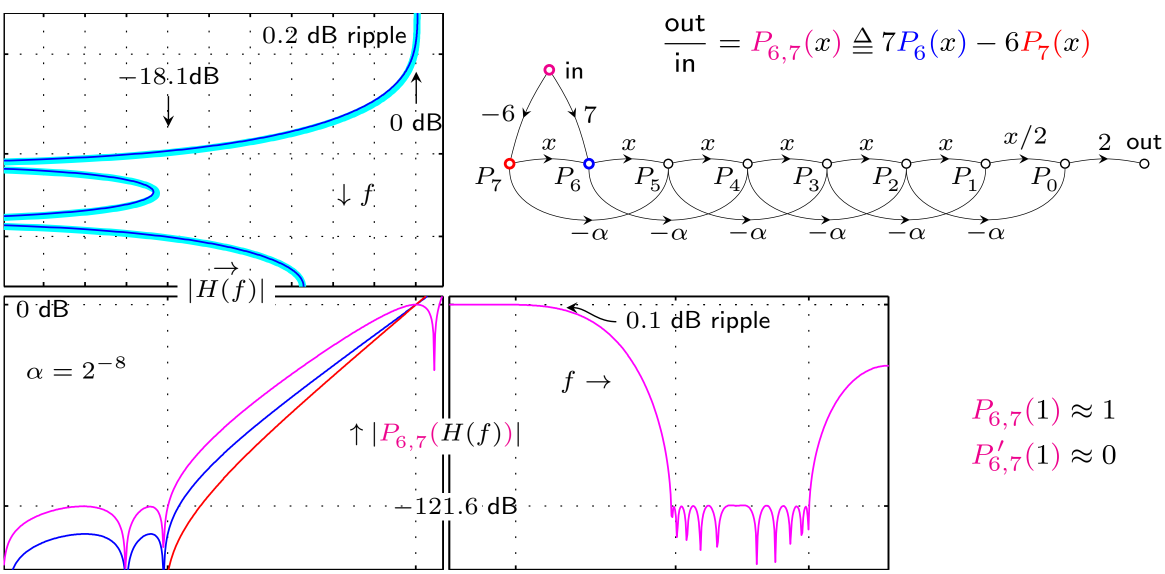
optimize $x = H(f)$
quantize to $h(n) = \big(-5 \quad 6 \quad 19 \quad 25 \quad 19 \quad 6 \quad -5\big)/64$



Passband flattening using a zero derivative in the transformation

optimize $x = H(f)$
quantize to $h(n) = (-5 \quad 9 \quad 15 \quad 27 \quad 15 \quad 9 \quad -5)/64$

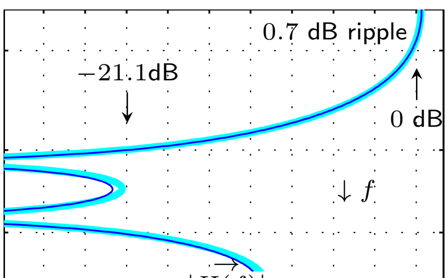
$$\frac{\text{out}}{\text{in}} = P_{6,7}(x) \triangleq 7P_6(x) - 6P_7(x)$$

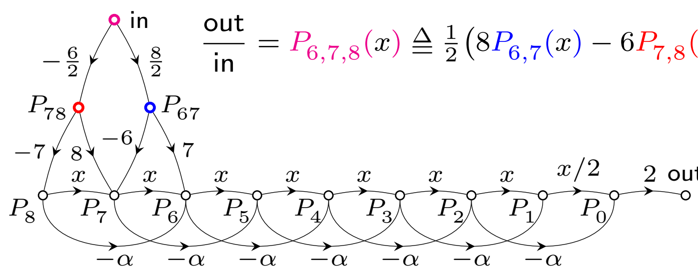


$P_{6,7}(1) \approx 1$
$P'_{6,7}(1) \approx 0$

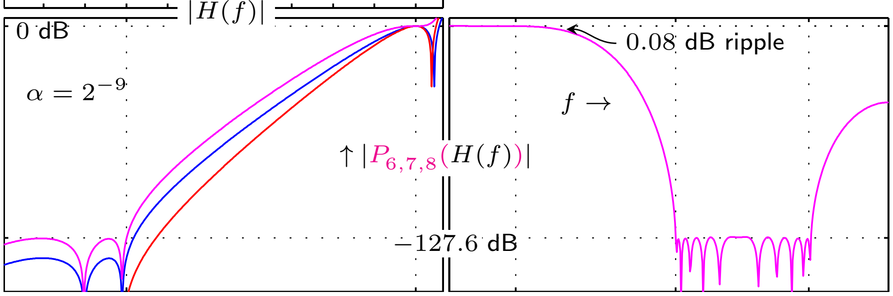
Passband flattening using two zero derivatives in the transformation

optimize $x = H(f)$
quantize to $h(n) = \big(-6 \quad 17 \quad 31 \quad 50 \quad 31 \quad 17 \quad -6\big)/128$





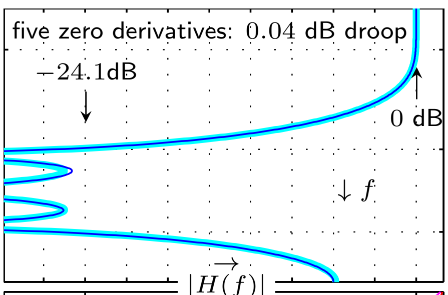
$$\frac{\text{out}}{\text{in}} = P_{6,7,8}(x) \triangleq \tfrac{1}{2}\big(8P_{6,7}(x) - 6P_{7,8}(x)\big)$$

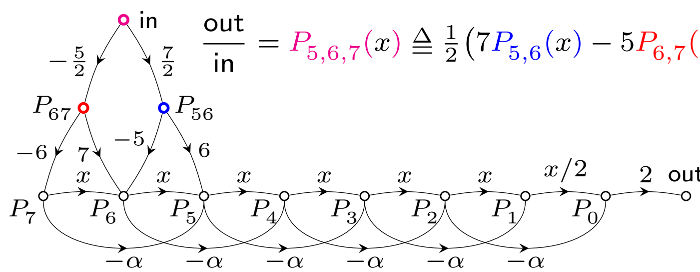


$P_{6,7,8}(1) \approx 1$
$P'_{6,7,8}(1) \approx 0$
$P''_{6,7,8}(1) \approx 0$

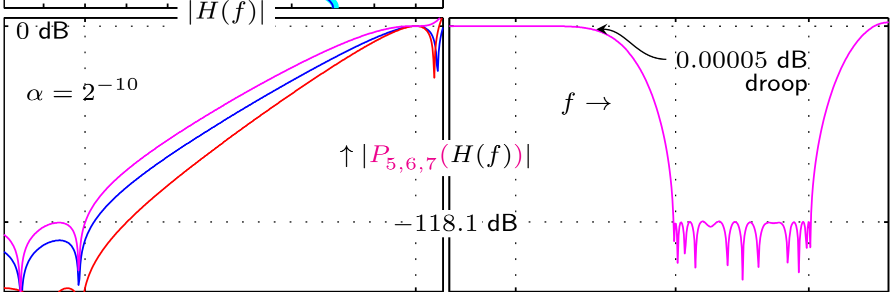
Extreme: zero two transformation derivatives and five prototype derivatives

optimize $x = H(f)$
quantize to $h(n) = \big(16 \;\; -41 \;\; 0 \;\; 20 \;\; 176 \;\; 170 \;\; 176 \;\; 20 \;\; 0 \;\; -41 \;\; 16\big)/512$





$$\frac{\text{out}}{\text{in}} = P_{5,6,7}(x) \triangleq \tfrac{1}{2}\big(7P_{5,6}(x) - 5P_{6,7}(x)\big)$$



$P_{5,6,7}(1) \approx 1$
$P'_{5,6,7}(1) \approx 0$
$P''_{5,6,7}(1) \approx 0$

Conclusions

- Arbitrarily deep equiripple stopbands and arbitrarily flat passbands.
- It is far from clear how best to obtain the most efficient (quantized) designs given tradeoffs among
  - the feedforward coefficient α,
  - the degree of the sharpening polynomial,
  - the derivative-zeroing input network, and
  - the design of the prototype filter.
- Can a sharpened filter be used as a prototype? Should it be?
- There are potential thesis topics here.